

Problem 1 Let $\Sigma = \{a, b, c, d\}$.

Let e be the regular expression defined as follows:

$$e = ab \cup cd \cup ac \cup bd \cup ad$$

Let L be the language defined by e .

In each of the cases below, state the cardinality of the given set. If this cardinality is finite, state the *exact number*. (An arithmetic expression is *not acceptable* as answer.) If this cardinality is infinite, state it and specify whether it is countable or uncountable.

(a) class of all languages over Σ

Answer:

infinite, uncountable

(b) class of languages over Σ that are regular

Answer:

infinite, countable

(c) class of languages over Σ that are not regular

Answer:

(d) L

Answer:

5

(e) $\mathcal{P}(\Sigma)$ (set of subsets of Σ)

Answer:

16

(f) $\mathcal{P}(L)$ (set of subsets of L)

Answer:

32

(g) class of languages over Σ that are finite

Answer:

infinite, countable

(h) class of languages over Σ that are infinite

Answer:

infinite, uncountable

(i) class of languages over Σ that have no finite description

Answer:

infinite, uncountable

(j) set whose regular expression over Σ is:

$$\emptyset \cup a$$

Answer:

1

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(k) set whose regular expression over Σ is:

$$\emptyset^* \cup a$$

Answer:

2

(l) set whose regular expression over Σ is:

$$\emptyset \cup e$$

Answer:

5

(m) set whose regular expression over Σ is:

$$\emptyset e$$

Answer:

0

(n) set whose regular expression over Σ is:

$$\emptyset \cup ee$$

Answer:

25

(o) set whose regular expression over Σ is:

$$\emptyset^* e$$

Answer:

5

(p) set whose regular expression over Σ is:

$$\lambda \cup e$$

Answer:

6

(q) set whose regular expression over Σ is:

$$\lambda e$$

Answer:

5

(r) set whose regular expression over Σ is:

$$\emptyset^* \cup \lambda^*$$

Answer:

1

(s) set whose regular expression over Σ is:

$$ae$$

Answer:

5

(t) set whose regular expression over Σ is:

$$e^*$$

Answer:

infinite, countable

(u) set whose regular expression over Σ is:

$$(ee)^*$$

Answer:

infinite, countable

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Problem 2 Let L be the language defined by the regular expression:

$$((caaa)^* \cup (ad \cup c^*bab)^* (bc^*a)) (dda)^*$$

Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, prove it.

Answer:

$$G = (V, \Sigma, P, S) \quad P: \begin{array}{l} S \rightarrow AB \\ A \rightarrow D | EF \\ D \rightarrow \Lambda | DD | caaa \\ E \rightarrow \Lambda | EE | ad | Jbab \\ J \rightarrow cJ | \Lambda \\ F \rightarrow bJa \\ B \rightarrow \Lambda | BB | dda \end{array}$$

$$\Sigma = \{a, b, c, d\}$$

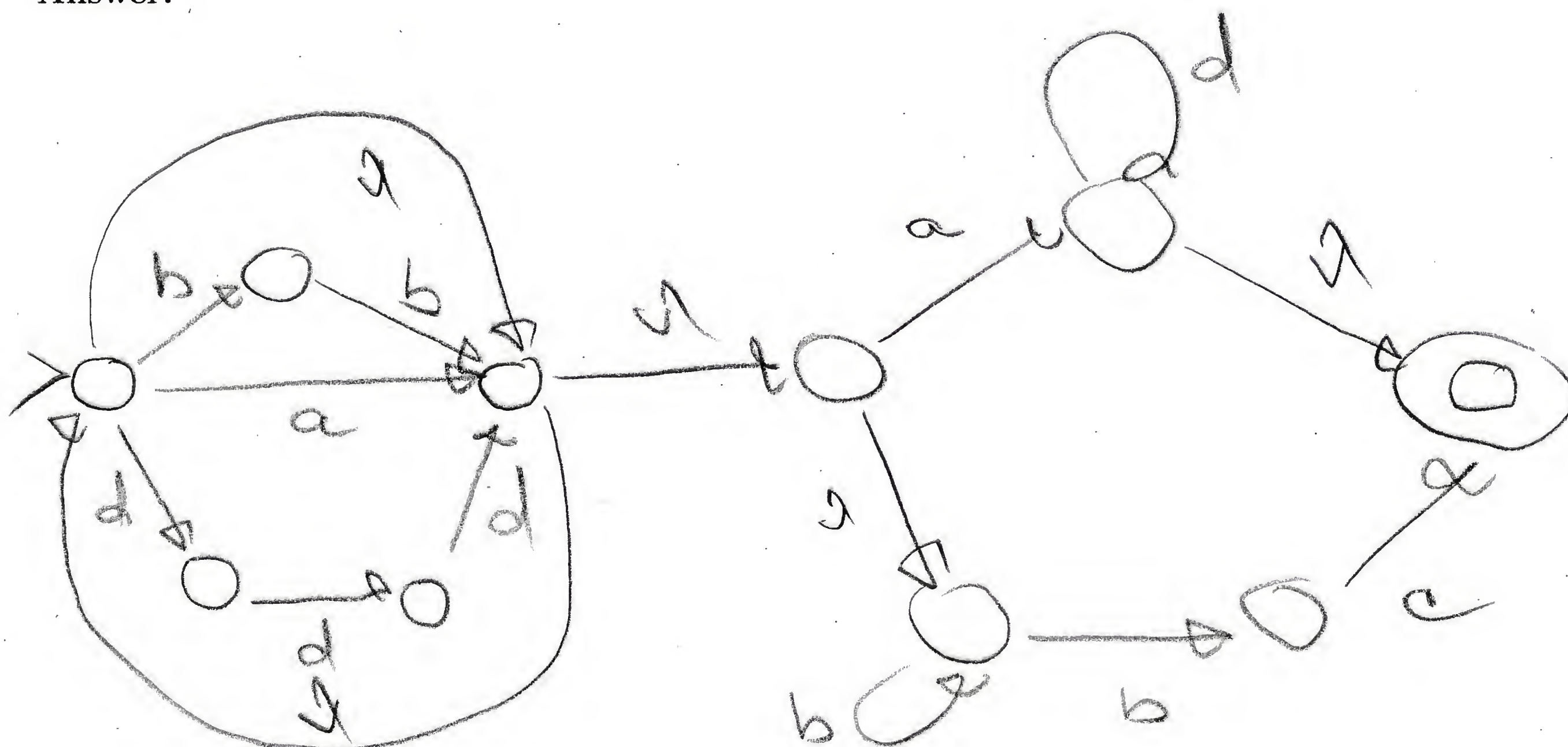
$$V = \{S, A, B, D, E, F, J\}$$

Problem 3 Let L be the language defined by the regular expression:

$$((a \cup bb \cup ddd)^* ((ad^*) \cup (b^*bc)))$$

Draw a state-transition graph of a finite-state automaton that accepts the language L . If such an automaton does not exist, prove it.

Answer:



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Problem 4 Let L be the set of exactly those strings over the alphabet $\Sigma = \{a, b, c\}$ that contain as substring at least one of the following three strings: cca , bac , ab .

Write a regular expression that defines L . If such a regular expression does not exist, prove it.

Answer:

$$(a|b|c)^*(cca \cup bac \cup ab)(a|b|c)^*$$

Problem 5 Let L be the set of exactly those strings over the alphabet $\Sigma = \{a, b, c\}$ whose length is odd, and the middle symbol is equal to the first symbol and to the last symbol.

Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, prove it.

Answer:

$$G = (V, \Sigma, P, S)$$

$$\Sigma = \{a, b, c\}$$

$$V = \{S, A, B, K, Z\}$$

$$P :$$

$$S \rightarrow aAa \mid bBb \mid cKc \mid a \mid b \mid c$$

$$A \rightarrow ZAZ \mid a$$

$$B \rightarrow ZBZ \mid b$$

$$K \rightarrow ZKZ \mid c$$

$$Z \rightarrow a \mid b \mid c$$

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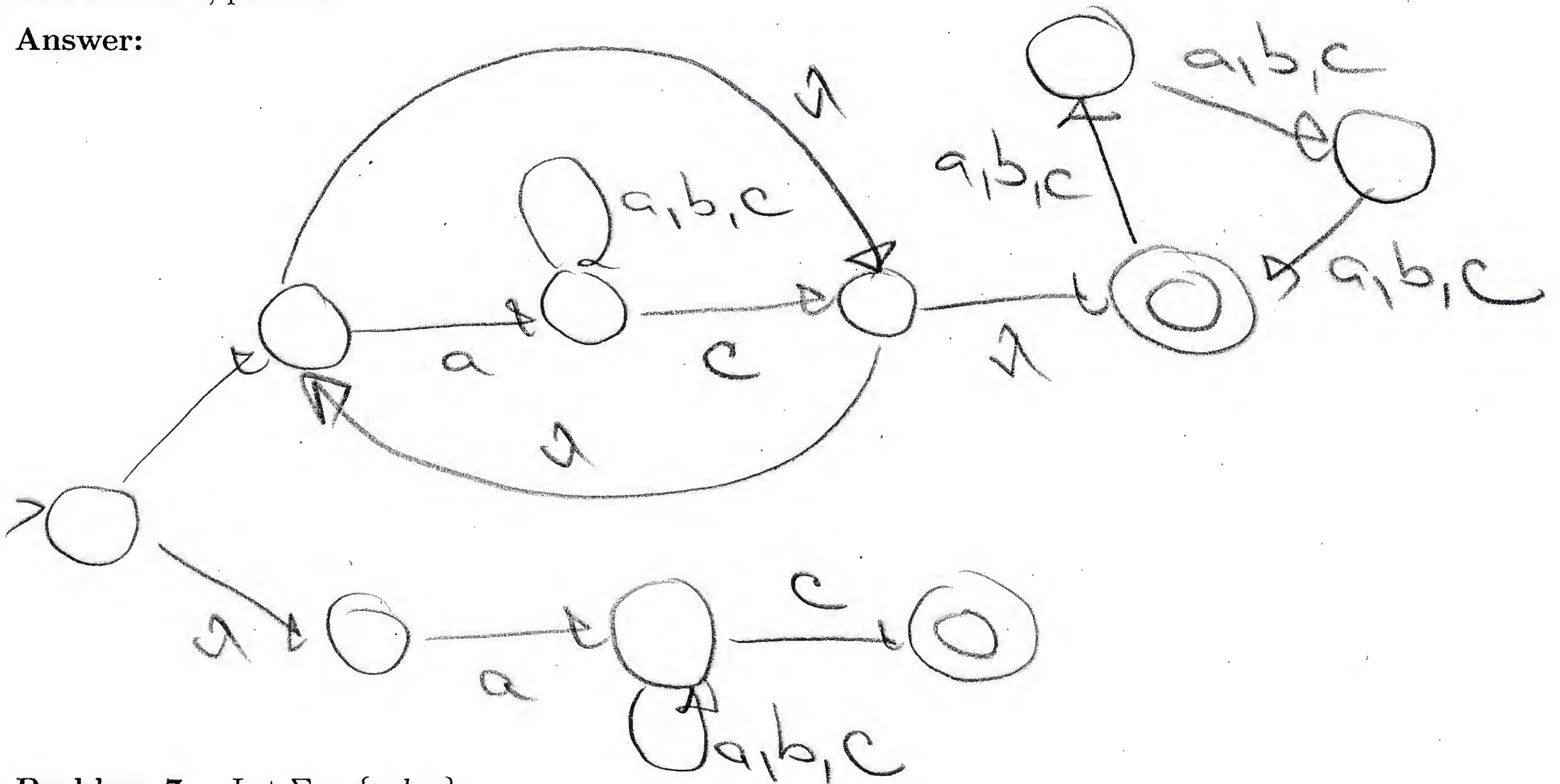
Problem 6 Let $\Sigma = \{a, b, c\}$.

Let L_1 be the set of exactly those strings over Σ that begin with a and end with c .

Let L_2 be the set of exactly those strings over Σ whose length is divisible by 3.

Draw a state-transition graph of a finite automaton that accepts the language $L_1^* L_2 \cup L_1$. If such an automaton does not exist, prove it.

Answer:



Problem 7 Let $\Sigma = \{a, b, c\}$.

Let L_1 be the set of exactly those strings over Σ where the number of c 's is equal to 3.

Let L_2 be the set of exactly those strings over Σ that contain (in any order) the two substrings: aac, bbc .

Write a complete formal definition of a context-free grammar that generates the language $L_1^* \cup L_2^*$. If such a grammar does not exist, prove it.

Answer:

$$\begin{array}{l}
 G = (V, \Sigma, P, S) \\
 \Sigma = \{a, b, c\} \\
 V = \{S, A, B, D, E\} \\
 P: \\
 S \rightarrow A \mid B \\
 A \rightarrow \epsilon \mid AA \mid DcDcDcD \\
 D \rightarrow \epsilon \mid DD \mid a \mid b \\
 B \rightarrow \epsilon \mid BB \mid EaacEbbcE \mid EbbcEaacE \\
 E \rightarrow \epsilon \mid EE \mid a \mid b \mid c
 \end{array}$$

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Problem 8 Let L be the language over the alphabet $\Sigma = \{a, b, c, d\}$ that contains exactly those strings whose form is:

$$\underbrace{b^i c^{j+3} a^\ell d^{m+2}} \underbrace{b^{n+1} a^{p+2} c^q}$$

where $i, j, \ell, m, n, p, q \geq 0$ are natural numbers such that: $i = m, n = q, j = \ell$

Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, prove it.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, S) \\ \Sigma &= \{a, b, c, d\} \\ V &= \{S, A, B, D, E\} \end{aligned}$$

$P:$

$$\begin{aligned} S &\rightarrow AB \\ A &\rightarrow bAd \mid Ddd \\ D &\rightarrow cDa \mid ccc \\ B &\rightarrow bBc \mid bE \\ E &\rightarrow aE \mid acc \end{aligned}$$

Problem 9 Let L be the language over the alphabet $\Sigma = \{a, b, c\}$ that contains exactly those strings which satisfy all of the following properties:

1. if the string does not contain any a 's, then the string is a concatenation of at least three palindromes whose length is odd and greater than 1;
2. if the string contains at least one a , then the string is palindrome whose length is odd and the middle symbol is a .

Write a complete formal definition of a context-free grammar that generates the language L . If such a grammar does not exist, prove it.

Answer:

$$\begin{aligned} G &= (V, \Sigma, P, S) \\ \Sigma &= \{a, b, c\} \\ V &= \{S, S_1, S_2, A, B, Z\} \end{aligned}$$

$P:$

$$\begin{aligned} S &\rightarrow S_1 \mid S_2 \\ S_1 &\rightarrow AS_1 \mid AAA \\ A &\rightarrow bAb \mid cAc \mid B \\ B &\rightarrow bZb \mid cZc \\ Z &\rightarrow b \mid c \\ S_2 &\rightarrow aS_2a \mid bS_2b \mid cS_2c \mid a \end{aligned}$$

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Problem 10 Let L be the language over the alphabet $\Sigma = \{a, b, c\}$ that contains exactly those strings which satisfy all of the following properties:

1. the three symbols a, b, c occur in the string in the alphabetic order;
2. the substring which consists of a 's is a non-empty palindrome whose length is even;
3. the substring which consists of b 's is a palindrome whose length is odd;
4. the substring which consists of c 's is not empty.

If L is regular, then use part (a) of the answer space below to write a regular expression that defines L , and do not write anything in part (b).

If L is not regular, then do not write anything in part (a) of the answer space, but complete the missing parts of the text given in part (b) so as to obtain a proof that L is not regular.

(a) Regular expression for L :

Answer:

$(aa)^*aa(bb)^*b c^*$

(b) Proof that L is not regular:

Observe that all words of L satisfy the following characteristic property:

Assume the opposite, that L is regular. Let π be the constant as in the Pumping Lemma for L . Let $w_0 \in L$ be a string defined as follows:

$w_0 =$

w_0 belongs to L because

w_0 must pump because

In any "pumping" decomposition of w_0 , the pumping window satisfies the following property:

because

By pumping _____ times, we obtain a string:

which violates the stated characteristic property because

and thus does not belong to L .

Since L violates the Pumping Lemma, L is not regular.